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31. $(23\frac{4}{7})^2=(\frac{165}{7})^2=(2\times11)\left(\frac{3\times5\times11}{2\times7}\right)(2\times5)\left(\frac{3}{2\times7}\right)=22\times\frac{11}{14}\times10\times\frac{3}{14}$. $\therefore$ Sides are $10\frac{3}{14}$, 12. and $21\frac{11}{14}$, and sides of $\triangle=330$ are $10\frac{3}{14}\sqrt{14}$, $12\sqrt{14}$, and $21\frac{11}{14}\sqrt{14}$.

32. $\triangle^2=22^2=2\times2\times11\times11=(2\times11)(2\times3\times3)(\frac{11}{3})(\frac{1}{3})=22\times18\times3\frac{2}{3}\times\frac{1}{3}$. $\therefore$ Sides are 4, $18\frac{1}{3}$. and $21\frac{2}{3}$, and sides of $\triangle=330$ are $4\sqrt{15}$, $18\frac{1}{3}\sqrt{15}$, and $21\frac{2}{3}\sqrt{15}$.

33. $\triangle^2=15^2=3\times3\times5\times5=(2\times5)(2\times3)(\frac{5}{2})(\frac{3}{2})=10\times6\times2\frac{1}{2}\times1\frac{1}{2}$. $\therefore$ Sides are 4, $7\frac{1}{2}$, and $8\frac{1}{2}$, and, of course, sides of $\triangle=330$ are $4\sqrt{22}$, $7\frac{1}{2}\sqrt{22}$, and $8\frac{1}{2}\sqrt{22}$.

34. $\triangle^2=10^2=2\times2\times5\times5=(3\times3)(5)\left(\frac{2\times5}{3}\right)(\frac{2}{3})=9\times5\times3\frac{1}{3}\times\frac{2}{3}$. $\therefore$ Sides are 4, $5\frac{2}{3}$, and $8\frac{1}{3}$, and sides of $\triangle=330$ are $4\sqrt{33}$, $5\frac{2}{3}\sqrt{33}$, and $8\frac{1}{3}\sqrt{33}$.

ALGEBRA.

79. Proposed by **C. W. M. BLACK, A. M.**, Professor of Mathematics, Wesleyan Academy, Wilbraham, Mass.

Of n persons A, B, C, etc., A gives to the others as much as each of them already has; then B gives to the others as much as each then has; and so on for each in turn. Finally, A, B, C, etc., have respectively a, b, c, etc., dollars. How much had each at first?

I. Solution by the PROPOSER.

Let $a+b+c+\text{etc}......=s$, which is constant. Consider the rth person, K, who has at the end k dollars. Let x=number of dollars he had at first. Before he gives away any, his original amount is doubled $r-1$ times and comes to equal $2^{r-1}x$. At that time all the rest must together have $s-2^{r-1}x$, and if he gives away that amount he has left $2^{r-1}x-(s-2^{r-1}x)=2^rx-s$. Afterwards his money is doubled $n-r$ times and finally equals $2^{n-r}(2^rx-s)=k$.

$\therefore x=(k+2^{n-r}s)/2^n$.

II. Solution by I. H. BRYANT, A. M., Instructor in Mathematics in Fort Smith High School, Fort Smith, Ark.; G. B. M. ZERR, A. M., Ph. D., Russell College, Lebanon, Va.; and J. SCHEFFER, A. M., Hagerstown, Md.

Let x_1, x_2, x_3, x_4, ..., x_n be the respective amounts each had at first.

Let $s=x_1+x_2+x_3+......+x_n=a+b+c+......(1)$.

$\therefore 2^{n-1}(2x_1-s)=a\ .....(2)$.

$2^{n-2}(4x_2-s)=b\ ......(3)$.

$2^{n-3}(8x_3-s)=c\ ......(4)$.

....

$(2^nx_n-s=n$.

(2), (3), (4), (5), etc., in (1) gives, $x_1=(a+2^{n-1}s)/2^n$, $x_2=(b+2^{n-2}s)/2^n$, $x_3=(c+2^{n-3}s)/2^n$, $x_4=(d+2^{n-4}s)/2^n$, ...$x_n=(n+s)/2^n$.

III. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

Let x_1, x_2, x_3, ... x_{n-1}, x_n=the money that A, B, C, etc., respectively, had at first. Put $x_n=r$, $x_{n-1}=2r-s$, $x_{n-2}=2x_{n-1}-s=4r-3s$, $x_{n-3}=2x_{n-2}-s=8r-7s$, etc., etc. The law that governs the forming of these values makes $x_1=2^{n-1}(r-s)+s$; $x_2=2^{n-2}(r-s)+s$; $x_3=2^{n-3}(r-s)+s$; $x_4=2^{n-4}(r-s)+s$;

etc., etc. Whence we deduce, $a=2^{n-1}[r-(n-1)s]$; $b=2^{n-2}\{r-[n-(2^2-1)]s\}$; $c=2^{n-3}\{r-[n-(2^3-1)]s\}$; $d=2^{n-4}\{r-[n-(2^4-1)]s\}$; etc., etc. But in order that all values are positive, we have the condition $r>(n-1)s$. Therefore, put $r=(n-1)s+t$, and we obtain,

$$\begin{array}{ll} a=2^{n-1}t, & x_1=2^{n-1}[s(n-2)+t]+s, \\ b=2^{n-2}(2s+t), & x_2=2^{n-2}[s(n-2)+t]+s, \\ c=2^{n-3}2(2^2-1)s+t, & x_3=2^{n-3}[s(n-2)+t]+s, \\ d=2^{n-4}2(2^3-1)s+t, \text{ etc.} & x_4=2^{n-4}[s(n-2)+t]+s, \text{ etc.} \end{array}$$

s and t may be any values, integral or fractional ; but s may also be *zero*. When $s=0$, the money each had at first equals, respectively, what he had at the end of the transaction ; or $a=x_1=2^{n-1}t$; $b=x_2=2^{n-1}t$; ......$x_{n-1}=2^2t$; $x_n=2t$. When $t=2s$, all have an equal sum at the end of the transaction ; or $2^ns=a=b=c$, etc. For obtaining integral values this form of solution is preferable.

IV. **Solution by F. M. McGAW, A. M., Professor of Mathematics, Bordentown Military Institute, Bordentown, N. J.**

This problem is concerned with a double n-fold series. Let A, B, C, etc., represent the amounts which each person has to start with ; then after the several exchanges, we have : A's nth term$=2^{n-1}[2A-\Sigma(A)]=a$; where $\Sigma(A)$ means $(A+B+C+\ldots\ldots$ to n letters) ; B's nth term equals $2^{n-2}[4B-\Sigma(A)]=b$; C's nth term$=2^{n-3}[8C-\Sigma(A)]=c$; and so on for R's nth term$=2^{n-r}[2^rR-\Sigma(A)]=\rho$; and N's nth term$=2^{n-n}[2^nN-\Sigma(A)]=\nu$.

Expanding, we may write,

$$\left.\begin{array}{l} (2^n-2^{n-1})A-2^{n-1}B-2^{n-1}C\ldots\ldots-2^{n-1}R\ldots\ldots-2^{n-1}N=a \\ -2^{n-2}A+(2^n-2^{n-2})B-2^{n-2}C\ldots\ldots-2^{n-2}R\ldots\ldots-2^{n-2}N=b \\ -2^{n-3}A-2^{n-3}B+(2^n-2^{n-3})C\ldots\ldots-2^{n-3}R\ldots\ldots-2^{n-3}N=c \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ -2^{n-r}A-2^{n-r}B-2^{n-r}C\ldots\ldots+(2^n-2^{n-r})R\ldots\ldots-2^{n-r}N=\rho \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ -2^{n-n}A-2^{n-n}B-2^{n-n}C\ldots\ldots-2^{n-n}R\ldots\ldots+(2^n-2^{n-n})N=\nu \end{array}\right\}\text{I.}$$

Whence by determinants,

$$\Delta.A=\left|\begin{array}{lllll} a, & -2^{n-1}, & -2^{n-1}, & \ldots.-2^{n-1}, & \ldots.-2^{n-1} \\ b, & 2^n-2^{n-2}, & -2^{n-2}, & \ldots.-2^{n-2}, & \ldots.-2^{n-2} \\ c, & -2^{n-3}, & 2^n-2^{n-3}, & \ldots.-2^{n-3}, & \ldots.-2^{n-3} \\ \ldots\ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ \rho, & -2^{n-r}, & -2^{n-r}, & \ldots.2^n-2^{n-r}, & \ldots.-2^{n-r} \\ \ldots\ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ \nu, & -1, & -1, & \ldots.-1, & \ldots..2^n-1 \end{array}\right|$$

Where Δ is the determinant of the coefficients.

Also solved by *H. C. WILKES.*

80. **Proposed by G. B. M. ZERR, A. M.,Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.**

Solve $1+x^4=a(1+x)^4$.